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EXTENSION OF FIXED POINT THEOREMS OF RHOADES USING ISHIKAWA ITERATION PROCESS

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In this paper we have discussed the extension of fixed point theorems of Rhoades using Ishikawa Iteration process. Suppose C be a non-empty subset of x, where x be a Banach space. And let T a mapping from C to itself. The iteration scheme called Ishikawa scheme is defined as follows :-

X_(n+1)=(1-?_n) X_n+?_n Ty_(n),n?0……..(I_1)
y_n=?_n Ty n+(1-?_n)Xn,n ?0……..(I_2)
………….X?C…… ………… ………..(I_3)

1. Introduction- In this paper it is shown that for mapping T which satisfy condition (A) or (B) above, if the sequence of Ishikawa iterates converges, it converges to the fixed point of T. These result extend the corresponding results of Rhoades [1] and Hicks and Kubicek [2]

2. Definition- 1. A mapping T : X ?X is a called a quesicontraction if there exists a constant k, O? K < 1 Such that for each X,Y ? X, where X be a Banach space,

?T_X-T_y ? ? k max {?x-y?,?x-T_x ?,?Y-T_y ?,?x-T_y ?,?Y-T_x ? }

Definition- 2. A mapping T : C ?C is called strictly pseuiocontractive constrictive if for k, O? K ? 1, and all X,Y ?-C, Where X be a normed linear space and C be a non empty subset of X.

?T_X-T_y ?2 ? ?x-y?+k ,?(I-T)X-(I-T)Y?2

Definition 2.1. T is called pseuioconstractive if for all X,Y ?C.

?T_X-T_y ?2 ??x-Y?2 +?(I-T)X-(I-T)Y?2

Definition 2.2. T is called demicontractive if for some k, O?K?1, for all X?C and Y? F(T) = {X?C: Tx = X},

?T_x-T_y ?2 ? ?X-Y?2+?X-T_x ?2

Definition 2.3 : T is called hemicontractive if for all X?C and Y?F(T)

?T_x-T_y ?2? ?X-T_x ?2+?X-Y?2

Definition 2.4 T is said to satisfy the condition (T) if for all X ?C and Y? F(T) F(T)

?T_X-y? ? ?x-y?

It is clear that any strictly pseuio contractive mapping is hemi contractive, any mapping satisfying condition (T) is demicontractive and a demicontractive mapping is hemicontractive but not conversely.

Theorem 1 - Suppose T : C ? C be a mapping satisfying (A), {Xn} the sequence of the Ishikawa scheme associated with T are such that {a_n} is bounded away from zero. It {x_n} converges to P, then P is a fixed point of T, where X be a normed linear space and C be a closed convex subset of X.

Proof - We have from (T1) that

X_(n+1)-X_n= ?_n (Ty_n-X_n)

Since X_n?P , ?x_(n+1)+1-x_n ??0. since {?_n} is bounded away from zero,

?Ty_n- x_n ? ?0 It also follows that ?P-Ty_n ? ?SinceT satisfies (A) we have

?Ty_n-Tx_n ?k max?{?y_n-x_n ?,?x_n-Tx_n ?,?y_n-Ty_n ?,?x_n-Ty_n ?+?y_n-Tx_n ? }

?y_n-x_n ?= ?(?_n Tx_n+(1-?_n) x_n-x_n ???_n ? x_n-tx_n)?

??X_n-Tx_n ? ? ?x_n-Tx_n ?+ ?Tx_n-Tx_n ? :

?y_n-Ty_n ? = ??_n T x_n T (1-?_n) X_n-Ty_n ??

?_n ?TX_n-Ty_n ?+ (1-?_n) ?Xn-Ty_n ?

? ?X_n-Ty_n ?+ ?TX_n-Ty_n ?;

?y_n-TX_n ?= ?_n TX_n+(1-?_n) X_n=TX_n?

? ?X_n-Ty_n ?+ ?Ty_n-TX_n ?,

Thus ?X_n-Ty_n ? ? 2K/(1-K) ?X_n-Ty_n ? We have by taking the limit as n ? ? ?TX_n-Ty_n ?? 0.

It follows that

?X_n-TX_n ? ? ?X_n-Ty_n ?+ ?Ty_n-TX_n ?

and ?P-Tx_n ? ? ?(P-x_n)? ?x_n-Ty_n ? ? 0

as n ? ? using the definition (1) of T and the triangle inequality, we have

?Tx_n-Tp? ? k max {?x_n-P?,?Xn-Tx_n ?,?X_n-P?+ ?X_n-Tx_n ?+ ?Tp-Tx_n ?,?P-Tx_n ?+ ?X_n-Tx_n ?+ Tx_n-Tp}

Thus we obtain, by taking the limit as n ? ?.

?Tx_n-T_P ??0

At last

?P-T_p ? ? ?p-Tx_n ?+ ?Tx_n-T_p ? ? 0

This means P = Tp.

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Opial's Lemma : Let H be a Hilbert space and the sequence {x_n} is weekly convergent to X_0 Then for any X?X_0 Lim ?X_n-X_0 ? ? Lim ?X_n-X?.

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